

St Michael's V.A. Junior Calculation Policy



This policy has been written to ensure consistency, progression and understanding throughout the school in the written methods that we teach. It contains the key written calculation procedures that will be taught within our school.

Long term aim

- have a secure knowledge of number facts and a good understanding of the four operations;
- be able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- For children to be able to select an efficient method of their choice (mental, with jottings or written) that is appropriate for the given task.

Expectations

- There is consistency and progression in the teaching and understanding of the four operations.
- Written methods tie in with and develop from mental methods.
- Jottings and mental methods are encouraged where appropriate.
- Estimating before carrying out calculations is always encouraged and can be used to check that an answer is reasonable.

The CPA Approach

One of the key learning principles in effective mathematics teaching is the concrete pictorial abstract approach, often referred to as the CPA approach. This is integral throughout a child's learning journey, from EYFS to Year 6.

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations. Within the mastery approach to teaching mathematics, it is commonly known that **presenting them together has the most positive impact on a child's learning.**

Concrete representation

The active stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

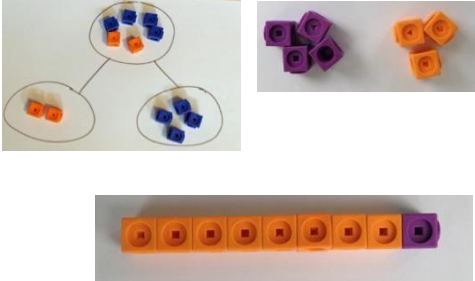

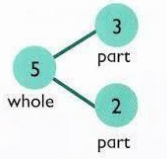

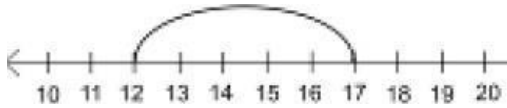
Pictorial representation

The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

Abstract representation

The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$ this is the ultimate mode, for it is clearly the most mysterious of the three.

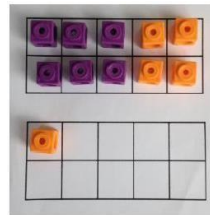
Addition

<u>Step</u>	<u>Strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
1	Combining two amounts.	<p>Use cubes or other objects to add two numbers together as a group or in a bar.</p> 	<p>Use pictures to add two numbers together as a group or as a bar.</p> 	<p>Use the part-part whole diagram as shown above to move into the abstract.</p>  <p>$4 + 3 = 7$ $10 = 6 + 4$</p>
2	Starting at the bigger number and counting on (number line strategy)	<p>Children use practical resources (e.g. bead strings or Numicon) to count on from a bigger number.</p> 	<p>Start at the bigger number on the number line and count on in ones or in one jump to find the answer.</p> 	<p>Children can identify the bigger number and count on from that (both mentally and/or on a number line).</p> <p>$5 + 12 = 12 + 5 = 17$</p>

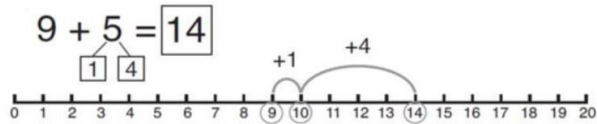
3

Regrouping
to make
ten

Find the biggest number. Count on to
make ten and then see what is remaining.

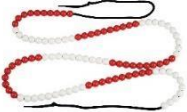
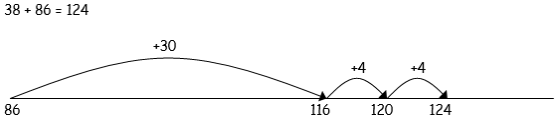

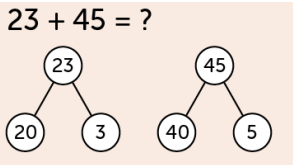


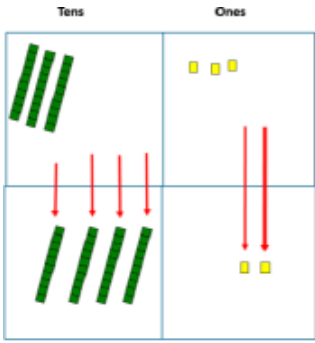
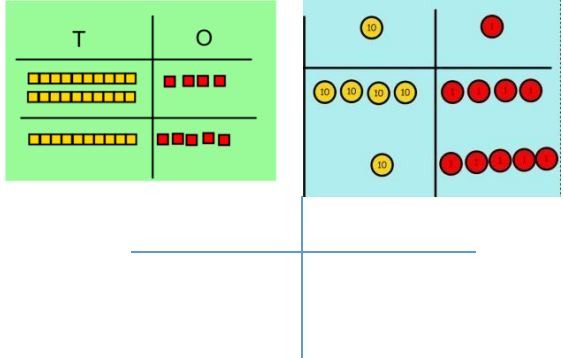
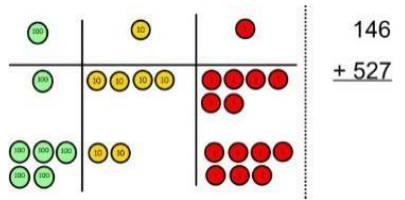
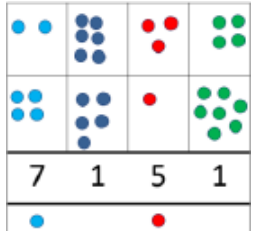
Use pictures of a number line.
Regroup by partitioning the smaller
number to then make 10 (bridge 10).




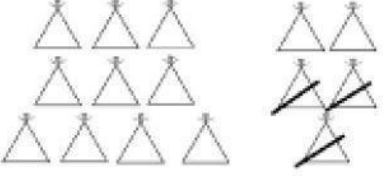
Mentally, partition
and add.
e.g. for $7 + 4$
If I am at seven,
how many more do I
need to make 10?
How many more do I
add on now?

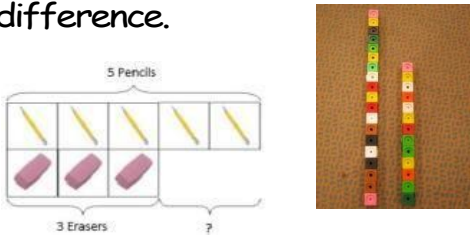
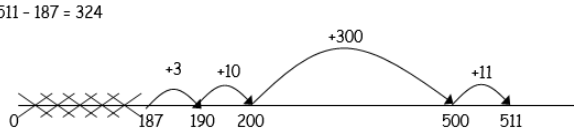

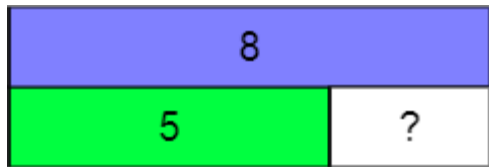
$$7 + 4 = 7 + 3 + 1 = 11$$

4	The empty number line	<p>Using concrete resources (e.g. bead string), start with the bigger number then jump to the nearest multiple of 10, then add the remaining amount.</p> 	<p>Count on from the largest number irrespective of the order of the calculation or use partitioning (only partitioning one of the numbers)</p> 	<p>Children are able to identify that they need to start with the biggest number, regardless of the way that the sentence is written.</p> $38 + 86 =$ $86 + 30 + 8 =$
5	<p>Partitioning.</p> <p>Adding the smallest value first.</p>	<p>Children use Dienes blocks. Partition each number. Add and then rearrange.</p> 	<p>Children partition numbers before adding them together</p>  <p>Add the ones: $3 + 5 = 8$ Add the tens: $20 + 40 = 60$ Answer: 68</p>	<p>Children partition numbers before adding them together.</p> $23 + 45 =$ $3 + 5 = 8$ $20 + 40 = 60$ $60 + 8 = 57$

6	Column method (no renaming)	<p>Add together the ones first then add the tens using Dienes and place value counters.</p> $\begin{array}{r} 33 \\ + 42 \\ \hline \end{array}$ 	<p>Children are able to draw pictorial representations of place value counters and Dienes, understanding the ones are added first.</p> $\begin{array}{r} 24 \\ + 15 \\ \hline \end{array}$ 	<p>Children can add two digits together, ensuring they start with the lowest value.</p> <div style="border: 1px dashed black; padding: 10px; background-color: #e0f2f7;"> <p style="text-align: center;"><u>Calculations</u></p> $21 + 42 =$ $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$ </div>
7	Column method (with renaming)	<p>Use Dienes then place value counters. Renamed digits are recorded below the line using the words 'rename 10 ones as 1 ten' or 'rename 10 tens as 1 hundred'.</p> 	<p>Children can draw a pictorial representation of the columns and Dienes or place value counters to further support their learning and understanding.</p> 	$\begin{array}{r} 146 \\ + 527 \\ \hline 673 \\ 1 \end{array}$

Subtraction

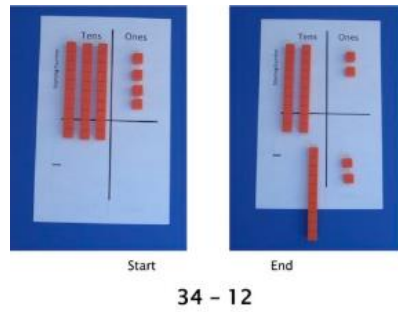
<u>Step</u>	<u>Strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
1	Taking away ones	Use physical counters, objects, cubes etc. to take away ones 	Cross out drawn objects to show what has been taken away.  $15 - 3 = \boxed{12}$	$18 - 3 = 15$ $8 - 2 = 6$

2	Using a number line	<p>Use objects to compare amounts by finding the difference.</p> 	<p>Find the difference by counting up from the smallest to the largest number. The number line may be displayed horizontally or vertically (like a thermometer). For example:</p> $511 - 187 = 324$ 	<p>Hannah has 19 grapes and Helen has 12 grapes. How many more grapes does Hannah have than Helen? Find the difference.</p> $19 - 12 = 7$
3	Part-part-whole model	<p>Link to addition -use the part whole model to help explain the inverse between addition and subtraction.</p> <p>What is the difference between 8 and 5. If 8 is the whole and 5 is one of the parts. What is the other part?</p> 	<p>Use the bar model to illustrate the calculation.</p> 	<p>Tom had a party. He made 8 cupcakes for his party. At the end of the party there were five left. How many were eaten?</p> $8 - 5 = 3$

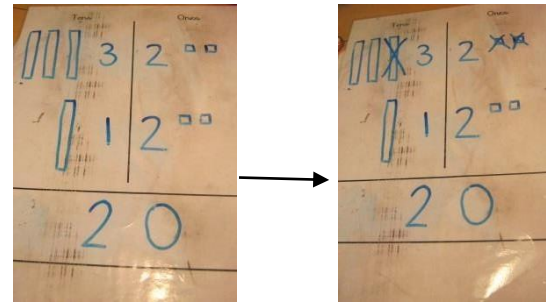
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Column method (no renaming)

Children should use Dienes to partition numbers to subtract.



Children should also draw Dienes or place value counters alongside their written methods to help to show their working, e.g. 32 - 12



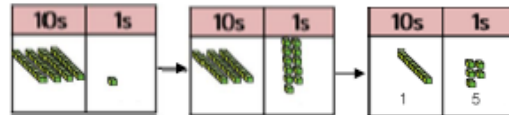
Children subtract numbers, starting with the smallest value.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

5 Column method (renaming)

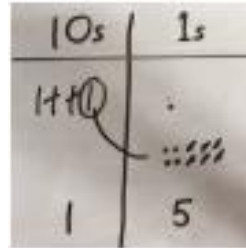
Children should use Dienes or place value counters. They should start by subtracting the ones, then the tens and so on. If they need to 'rename', children must swap their counters, e.g. swap 1 ten for 10 ones and place these in the ones column.

41 - 26

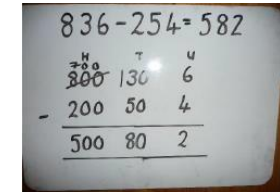


Represent the Dienes pictorially remembering to show any exchanges.

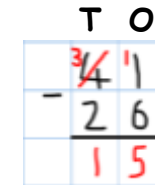
41 - 26



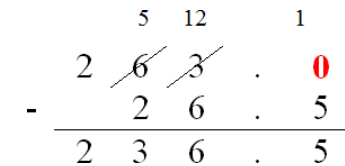
Some children may begin by using the expanded method, but the majority will begin with the short method.



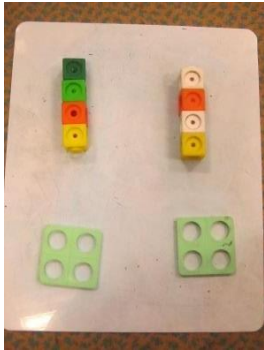

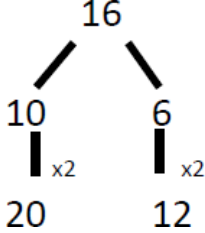
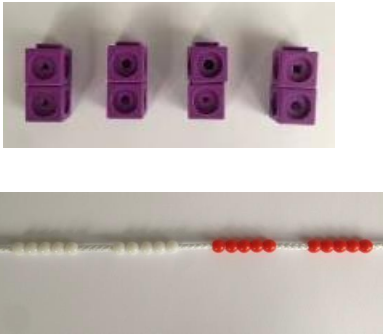
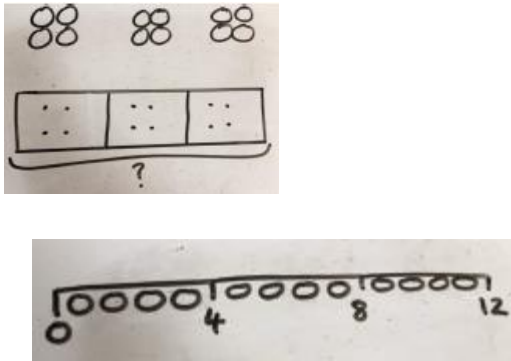

Short method:



This also applies for decimals:



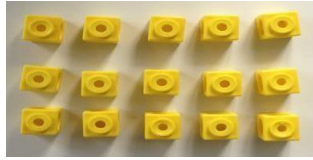
Multiplication

Step	Strategies	Concrete	Pictorial	Abstract
1	Doubling	<p>Use practical resources to double a range of numbers.</p> 	<p>Draw pictures to show how to double numbers.</p> <p style="text-align: center;">Double 4 is 8</p> 	<p>For numbers greater than 10, partition the number and then double each part before combining it back together.</p> 
2	Counting in multiples/ Repeated addition	<p>Count in multiples supported by concrete objects in equal groups.</p> 	<p>Use a numberline or pictures to continue supporting counting in multiples. E.g. 3×4</p> 	<p>3×4 is 3 times 4 is $4 + 4 + 4 = 12$ or 3 lots of 4</p> <p>3×4 on empty numberline:</p> 

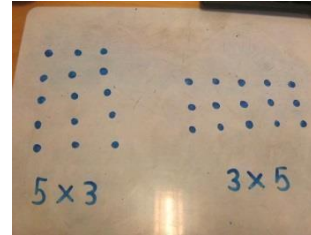
3

Arrays

Create arrays using counters and cubes to show multiplication sentences.



Draw arrays (in different rotations to show commutativity). Links should be made between arrays and the area of rectangles.



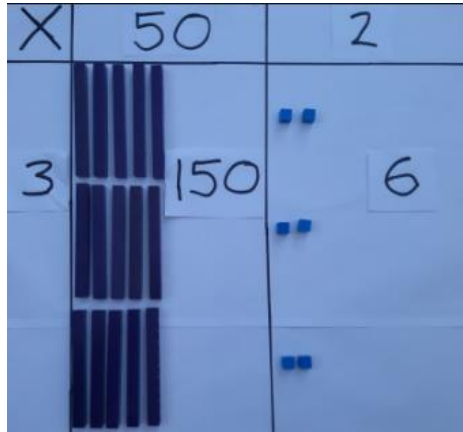
Use an array to write multiplication sentences and reinforce repeated addition.

$$\begin{aligned} 5 \times 3 &= 15 \\ 3 \times 5 &= 15 \\ 5 + 5 + 5 &= 15 \\ 3 + 3 + 3 + 3 + 3 &= 15 \end{aligned}$$

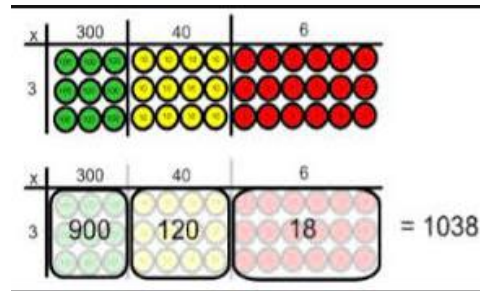
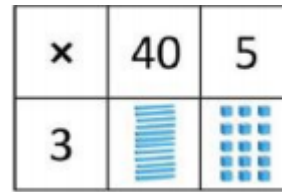
4

Grid method

Use Dienes or place value counters to represent multiplications in a grid. Link to arrays.
e.g. 52×3



Children draw their method using Dienes or place value counters.



Partition digits and multiply the smallest value first.

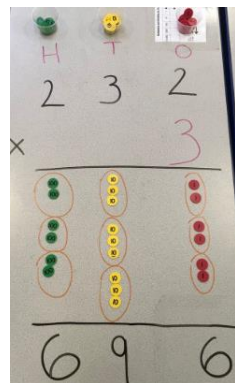
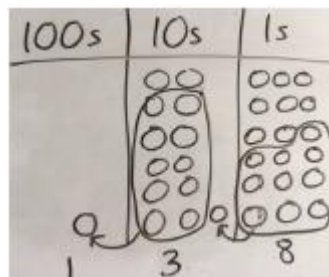
$$23 \times 7 = 161$$

x	20	3	Total
7	140	21	161

The grid is adjusted according to the digits, e.g. 2 digit x 2 digit = 2 x 2 grid

5

Column multiplication

Children use Dienes or place value counters, e.g. 6×23 Children draw their method using Dienes or place value counters, e.g. 23×6 

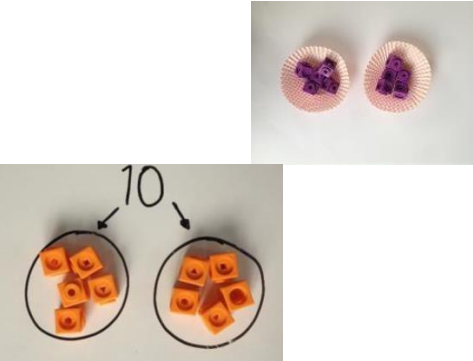
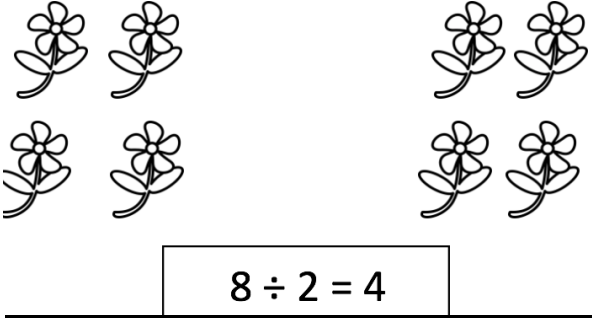
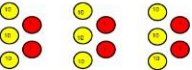
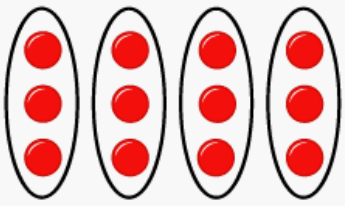
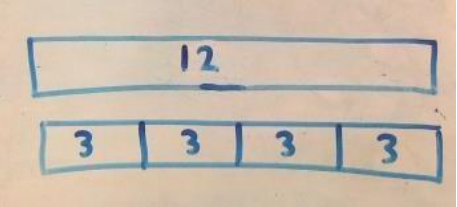
Children can start with the expanded column method.

$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$

Then move on to formal abstract method.

$$\begin{array}{r}
 1 2 4 \\
 \times 2 6 \\
 \hline
 7 4 4 \\
 2 4 8 0 \\
 \hline
 3 2 2 4 \\
 \hline
 1 1
 \end{array}$$

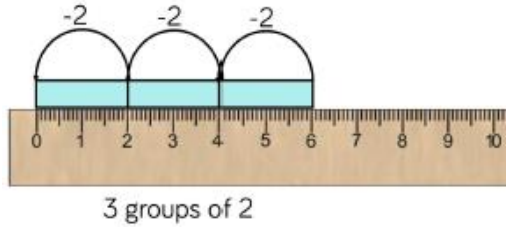
Division

Step	Strategies	Concrete	Pictorial	Abstract
1	Sharing objects into groups (to include halving)	Sharing a range of objects into a variety of groups 	Representing the sharing of objects pictorially.  $8 \div 2 = 4$	Partitioning the number into equal groups. $8 \div 2 = 4$
2	Division by grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. With larger numbers: $96 \div 3 = 32$ 	Divided into groups pictorially:  Show equal groups using a bar model: $12 \div 4 = 3$  Use a numberline to show equal groups	Divide into equal groups. $12 \div 4 = 3$ How many are in each group?

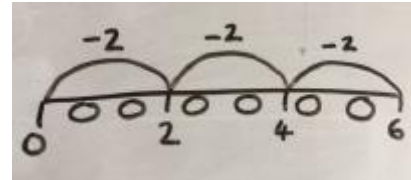
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Repeated subtraction on a number line

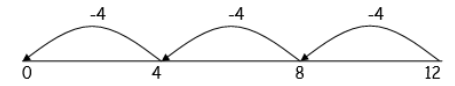
Use a range of resources to repeatedly subtract equal groups.
e.g. $6 \div 2$ using Cuisenaire above a ruler



Use an empty number line with pictures to support repeated subtraction.
 $6 \div 2$



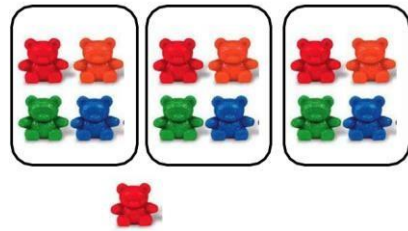
Use an empty number line for repeated subtraction, e.g. $12 \div 4$



4

Division with a remainder (leading onto chunking)

$13 \div 3 =$
Divide objects between groups and see how much is left over

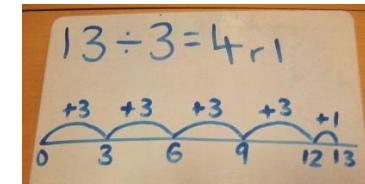


Divide into equal groups using drawings, clearly showing the remainder, e.g. $14 \div 4$



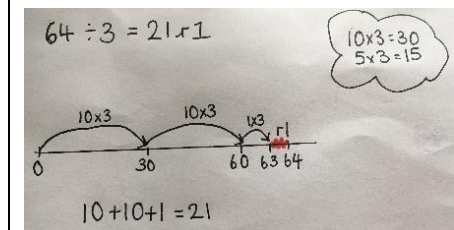
Complete written divisions and use the shorthand 'r' for remainder

Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



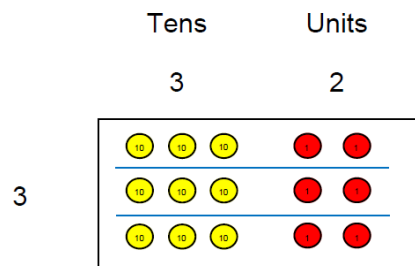
Chunking:

Jump forward in chunks (groups of multiples) on a number line and then see how many more you need to jump to find a remainder.

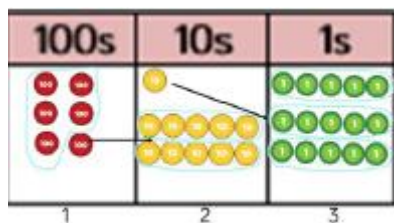


5 Short division

Short division (no exchanging) using Dienes or place value counters in the bus stop method, e.g. $96 \div 3 =$

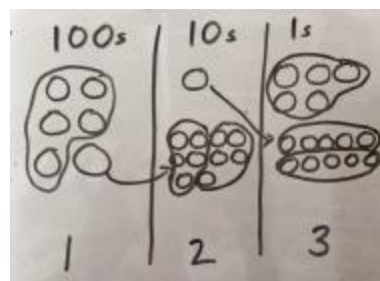


Short division (with exchanging) using Dienes or place value counters in the bus stop method, e.g. $615 \div 5 =$

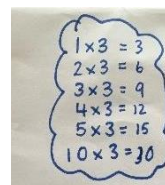


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters? (1)
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters? (2)
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 one counters? (3)

Represent the place value counters or Dienes pictorially, e.g. $615 \div 5 =$



Some children may find it beneficial to note down key multiples of the number that they are dividing by in a 'thought bubble'.



Written bus stop method

$$5 \overline{) 615} \begin{matrix} 123 \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{matrix}$$

6

Long division (Years 5/6 only)

<p>1. Divide.</p> $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	<p>2. Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	<p>3. Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ -2 \downarrow \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
<p>Divide.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	<p>Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	<p>Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
<p>1. Divide.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	<p>2. Multiply & subtract.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	<p>3. Drop down the next digit.</p> $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>